Table 1 Comparison of chemically and nuclear-explosive-propelled interceptors[a]

	Chemical	Nuclear
Specific impulse	500 s	42,500 s
Mass ratio, M_i/M_F	3.44	7.19
Initial mass	6,202 tons	3.35 tons
Final mass	1,803 tons	466 kg
Rocket velocity	$6.06 \text{ km} \cdot \text{s}^{-1}$	$821 \text{ km} \cdot \text{s}^{-1}$
Intercept range	293 Mm	14.6 Gm
Intercept time	5.6 days	5 h[c]
Collision energy	8.67×10^{21} erg[b]	1.61×10^{20} erg
	207 kT H.E.	38 kT H.E.
Blow-off mass	2.21 MT	486 kT
Fraction ejected, M_e/M_a	17.6%	3.86%

[a]Assumes 100-m radius asteroid with density $3 \text{ g} \cdot \text{cm}^{-3}$; mass $M_a = 12.6$ MT; velocity $v = 25 \text{ km} \cdot \text{s}^{-1}$. Crater parameters: $\beta = 0.9$ and $\alpha = 2 \times 10^{-4}$ $\text{gm}^{(1-\beta)/2} \cdot \text{cm}^{-\beta} \cdot \text{s}^{\beta}$. Thirty percent energy to blow-off ($\delta = 0.775$).
[b]Collision will probably cause asteroid to break up.
[c]You can shoot more than once.

the first malfunction. From Eq. (3), the mass of the ejecta is about 4.86×10^{11} gm or about 3.86% of the asteroid's mass. The interceptors would most likely be stationed at an Earth–moon Lagrange point, so the fission products from the nuclear-explosive propellant would be dispersed well outside the Earth's magnetosphere.

Table 1 compares the chemically and nuclear-explosive-propelled interceptors if launched when the asteroid is 1/10 a.u. from Earth. If detected at that range, about a week would remain before the assailant collides with the Earth. The chemically propelled interceptor would have only one chance.

Conclusions

The effectiveness of using nuclear-explosive-propelled interceptors derives mainly from the fact that the interception and deflection occur farther from the Earth. The numbers used for the example are somewhat arbitrary, but the essential conclusions hold over a broad range of assumptions. The interceptor deflects the incoming object by kinetic energy alone, which might make it more politically acceptable than one bearing a nuclear warhead.

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Modeling the Short-Term Evolution of Orbital Debris Clouds in Circular Orbits

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Nomenclature

a = semimajor axis of orbit, km
c = maximum cross-track dimension of cloud, km
d = maximum radial dimension of cloud, km
E = orbital energy per unit mass, km^2/s^2
e = eccentricity of fragment orbit
i = inclination of orbit, radians
k_m = series of Fourier constants
n = mean motion of orbit, rev/s
r = radial vector, km
V = volume, km^3
v = inertial velocity, km/s
Δv = maximum velocity impulse imparted to fragments, km/s
θ = true anomaly from point of breakup, radians

Subscripts

$-\Delta v$ = specific orbital energy less than parent satellite
$+\Delta v$ = specific orbital energy greater than parent satellite

Introduction

THE breakup of a satellite in orbit will result in the formation of a debris cloud. In the absence of perturbations such as atmospheric drag and gravitational anomalies, this cloud takes the form of a torus after several days. The torus has a pinch point at $\theta = 2m\pi$ ($m = 0, 1, 2, \ldots$) and a pinch wedge at $\theta = (2m + 1)\pi$, where θ is the angular displacement from the point of breakup. The torus defines the maximum extent of the cloud envelope in inertial space and is completed when the extremes of the cloud (relating to the fragments with the greatest and least orbital energies E and denoted by $\theta_{-\Delta v}$ and $\theta_{+\Delta v}$, respectively, in Fig. 1) extend 2π away from the parent satellite locus (the position in inertial space that the satellite which breaks up would have occupied had it remained intact). In this Note, we derive a new analytic model for the evolution of the debris cloud under such conditions.

Current Modeling

The Chobotov model,[1] based on the linear relative motion in a circular orbit, predicts the cloud volume following an isotropic breakup by an expression of the form:

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$$V(\theta) = 4\pi/3(\Delta v/\dot{\theta})^3\,|\sin\theta\,[-3\theta\sin\theta + 8(1-\cos\theta)]| \qquad (1)$$

If we consider an orbital breakup from an initial circular orbit of 1500 km (mean motion $n \approx 12.42$ rev/day), and choose a limiting breakup velocity $\Delta v = 250$ m/s, then $\dot{\theta} = 2\pi n \approx 9.03 \times 10^{-4}$ radians/s and $(\Delta v/\dot{\theta})^3 = 2.1 \times 10^7$ km^3. The instantaneous volume of the cloud predicted by this model is shown in Fig. 2. The cloud takes the form of a pulsating ellipsoid with zero volume at $\theta = m\pi$. This is a problem because we know intuitively that as the fragments have different orbital periods, the time taken for them to return to the point of breakup will vary. This implies that the cloud will be finite in volume at any time following breakup and not zero as suggested by the Chobotov solution.

Mathematical Development

Our approach is to treat the evolving cloud as two limbs, one which advances ahead of the parent satellite locus and one which retreats behind it. We assume that the advancing limb contains fragments whose orbital energy per unit mass E are less than the parent satellite. The fragments in the retreating limb are assumed to have specific orbital energies greater than the parent satellite. Assuming that the cross section of the limb in the along-track direction can be represented by a semiellipse of area $\pi/2\, c\, \psi_1(\theta)\, d\, \psi_2(\theta)$, then the volume of the limb envelope is given by

$$V(\theta) = \frac{\pi a c d}{2}\int_{\theta_0}^{\theta_1} \psi_1(\theta)\,\psi_2(\theta)\,d\theta \qquad (2)$$

where a is the semimajor axis of the parent satellite orbit, c and d are the *maximum* cross-track and radial dimensions of the torus, and θ_1 and θ_0 are the angles swept out by the ends of the limb as it rotates in an inertial frame.

Where $\psi_1(\theta)$ takes the form $|\sin\theta|$ and $\psi_2(\theta)$ takes the form $|\sin\theta/2|$ to ensure that 1) a pinch point occurs at $\theta = m\pi$; 2) a pinch wedge occurs at $\theta = (2m+1)\pi$; 3) the maximum radial dimension of the limb occurs at $\theta = (2m+1)\pi$; and 4) the maximum cross-track dimension of the limb occurs at $\theta = (m + \frac{1}{2})\pi$. If we take the Fourier series representation of $|\sin\theta|$, then the product of the functions $\psi_1(\theta)$ and $\psi_2(\theta)$ can be integrated with respect to θ to give

$$\int \left|\sin\frac{\theta}{2}\right| \,|\sin\theta|\, d\theta = \frac{4}{\pi^2}\left[k_0\,\theta + \sum_{m=1}^{\infty}(k_m \sin m\theta)\right] \qquad (3)$$

where the significant k_m terms take the following values in all cases: $k_0 = 1.044$, $k_1 = -0.422$, $k_2 = -0.396$, $k_3 = 0.070$, $k_4 = -0.021$, $k_5 = 0.016$, and $k_6 = -0.008$.

The radial cloud dimension is derived from the change in the orbital energy E due to an impulsive perturbation tangential to the parent satellite orbit:

$$E_1 - E = 1/2(v_1^2 - v^2) = -(\mu/2)\,[1/a_1 - 1/a] \qquad (4)$$

where μ (= 398600.4 km^3/s^2) is the product of the universal gravitational constant and the mass of the Earth. The greatest change in energy is achieved when the maximum velocity impulse Δv acts in the same or opposite direction to the satellite velocity vector. These two bounding cases are denoted by the subscripts $+\Delta v$ and $-\Delta v$, respectively. Introducing the parameters γ and λ:

$$\begin{aligned} a/a_{+\Delta v} &= 2 - \gamma, \qquad \gamma = (1 + (\Delta v/v))^2 \\ a/a_{-\Delta v} &= 2 - \lambda, \qquad \lambda = [1 - (\Delta v/v)]^2 \end{aligned} \qquad (5)$$

The maximum radial dimensions of the respective limbs are then given by $d_{+\Delta v} = 2a_{+\Delta v} - 2a$ and $d_{-\Delta v} = 2a - 2a_{-\Delta v}$.

The maximum cross-track displacement, which is common to both limbs, is given by

$$c = a\Delta i \qquad (6)$$

where Δi is the change in inclination produced by the maximum velocity impulse acting normal to the parent satellite orbit plane. This is given by the law of cosines as

$$\Delta i = \cos^{-1}\beta = \cos^{-1}[1 - 1/2(\Delta v/v)^2] \qquad (7)$$

The angular displacement of the limb extremes $\theta_{-\Delta v}$ and $\theta_{+\Delta v}$ can be expressed in terms of θ. From the equation of the center,[2]

$$\theta_{-\Delta v} = \theta(2-\lambda)^{3/2} - (2e_{-\Delta v} - \tfrac{1}{4}e_{-\Delta v}^3)\sin\theta(2-\lambda)^{3/2} + \tfrac{5}{4}e_{-\Delta v}^2 \sin 2\theta(2-\lambda)^{3/2} + \mathcal{O}(e_{-\Delta v}^3) \qquad (8)$$

$$\theta_{+\Delta v} = \theta(2-\gamma)^{3/2} + (2e_{+\Delta v} - \tfrac{1}{4}e_{+\Delta v}^3)\sin\theta(2-\gamma)^{3/2} + \tfrac{5}{4}e_{+\Delta v}^2 \sin 2\theta(2-\gamma)^{3/2} + \mathcal{O}(e_{+\Delta v}^3) \qquad (9)$$

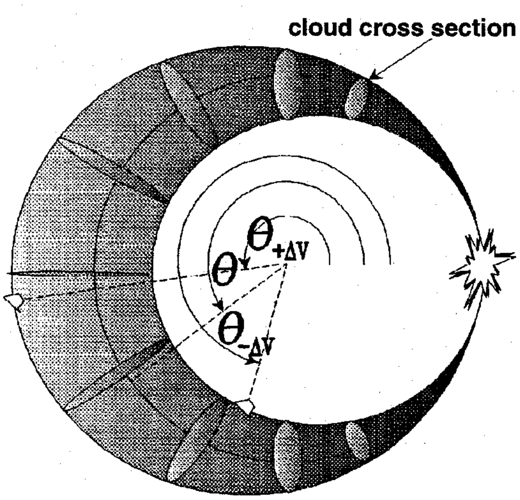

Fig. 1 Limiting torus.

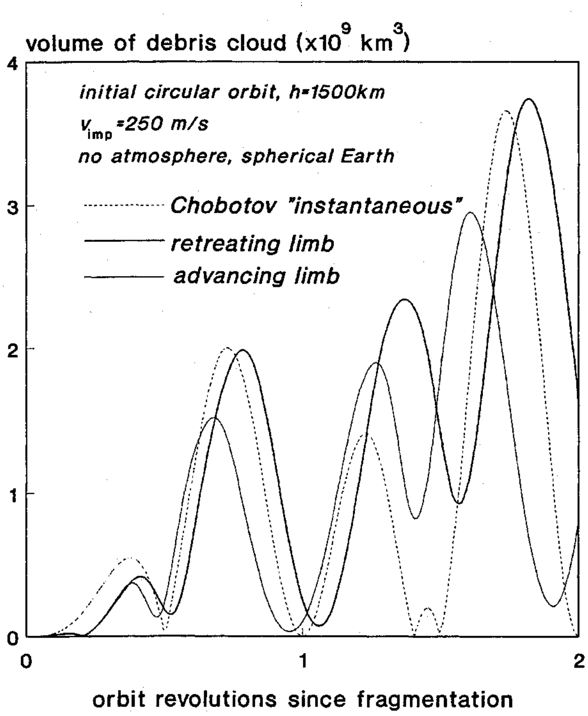

Fig. 2 Comparison of volume predictions.

The respective eccentricities are derived from consideration of angular momentum h:

$$h_1^2 - h^2 = |r \times v_1|^2 - |r \times v|^2 = \mu a_1(1 - e_1^2) - \mu a \quad (10)$$

$$e_{-\Delta v} = \sqrt{1 - \lambda(2 - \lambda)}, \qquad e_{+\Delta v} = \sqrt{1 - \gamma(2 - \gamma)} \quad (11)$$

Finally, the volumes of the advancing and retreating limbs are given by

$$V_{-\Delta v}(\theta) = \frac{4}{\pi} a^3 \left(\frac{1 - \lambda}{2 - \lambda}\right) \cos^{-1} \beta \times \left[k_0(\theta_{-\Delta v} - \theta) + \sum_{m=1}^{\infty} (k_m [\sin m\theta_{-\Delta v} - \sin m\theta]) \right] \quad (12a)$$

$$V_{+\Delta v}(\theta) = \frac{4}{\pi} a^3 \left(\frac{\gamma - 1}{2 - \gamma}\right) \cos^{-1} \beta \times \left[k_0(\theta - \theta_{+\Delta v}) + \sum_{m=1}^{\infty} (k_m [\sin m\theta - \sin m\theta_{+\Delta v}]) \right] \quad (12b)$$

Preliminary Comparison with the Chobotov Model

Assuming the same initial conditions used to illustrate the Chobotov model (fragmentation at an altitude of 1500 km and a limiting impulse velocity of 250 m/s), the limb volumes predicted by the torus model are plotted in Fig. 2. It is apparent that the new model represents the pulsating nature of the cloud induced by the pinch regions. Unlike the Chobotov model, however, the volumes predicted by the torus model do not return to zero at $\theta = m\pi$ but remain finite. The torus model also predicts that the volume of the retreating limb will be larger than that of the advancing limb. Shortly after breakup the torus model predicts that the limb volumes will reduce to zero. This is due to the cloud effectively turning itself inside out as high-energy fragments are overtaken by low energy fragments. Finally, we see that the new model predicts that the maxima and minima of cloud volume will occur at different times as compared to the Chobotov model.

Conclusions

A new model for the short-term evolution of a debris cloud resulting from the isotropic breakup of a satellite in an initial circular orbit has been developed. In a similar manner to the classical Chobotov model, the cloud volume can be determined given only the initial altitude of the parent satellite and the maximum velocity impulse imparted to the fragments. The new model does not suffer from the singularities at the pinch points observed with the Chobotov model. In addition, the new model predicts that the volume will decrease at locations other than at the pinch points. This behavior was predicted by the analysis of Hujsak[3] using nonlinear dynamical models. Further work is necessary to compare the results derived from this new approach with more deterministic cloud evolution models in order to verify that the predicted density *hot spots* occur at the correct locations and that the predicted volumes at these points are representative.

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